(3 Hours) **[Total Marks: 60]**

N.B.: (1) Attempt any four questions.
(2) Figures to the right indicate full marks.
(3) Simple non-programmable calculator is allowed.

Q.1 (a) State and prove Jordan decomposition theorem. **(09)**

(b) In a sequence of Bernoulli trials if p_n is probability of odd no. of successes on n^{th} trial . Find i) generating function of p_n ii) p_n. **(06)**

Q.2 (a) i) Define joint distribution function (d.f) of (X,Y). **(05)**
ii)Examine whether following is a joint d.f

$$F(x,y) = \begin{cases} 0 & x < 0, y < 0, x + y < 1 \\ 1 & x + y \geq 1, x > 0, y > 0 \end{cases}$$

(b) Given joint p.d.f of r.v (X,Y) **(10)**

$$f(x,y) = \begin{cases} \frac{\beta^{\alpha+\gamma} x^{\alpha-1}}{\Gamma\alpha\ \Gamma\gamma} (y-x)^{\gamma-1} e^{-\beta y} & 0 < x < y < \infty \end{cases}$$

Find i) conditional prob. distribution of Y given X=x.
ii)prob .distribution of Y-X.

Q.3 (a) X and Y are independent Poisson r.v.s with parameters λ_1 , λ_2 $respectively$.Derive the conditional distribution of X given X+Y. **(05)**

(b) (i) Define compound distribution. **(10)**
ii) X_1 , X_2,…,X_N are i.i.d.r.v.s. Show that $E(S_N)=E(N)E(X)$ where $S_N= X_1 + X_2+…+X_N$ and N is a r.v.
iii)X_1 , X_2,…X_N are N independent r.v with common distribution exponential with mean 1. If N has p.m.f. $P[N=n]=pq^{(n-1)}$, n=1,2,…
Obtain distribution of $S_N= X_1 + X_2+…+X_N$

Q.4 (a) i)Suppose the r.v. X follows $U(-\frac{\pi}{2},\frac{\pi}{2})$ distribution. Obtain the distribution of Y=tanX. **(09)**
ii)Comment on moments of Y .
iii)Find the distribution of 1/Y.

(b) Suppose the r.v. X has beta distribution of first kind with parameters (α, β). Show that $P[X \leq p] = P[Y \geq \alpha]$, where Y has Binomial distribution with parameters ($n = \alpha + \beta - 1, p$). **(06)**

Q.5 (a) i)Define joint moment generating function(m.g.f) of (X,Y). **(07)**

ii)State relation between joint and marginal m.g.fs .

iii) Given p.d.f of the r.v. X as

$$f(x) = \frac{1}{2\lambda} e^{\frac{-|x-\mu|}{\lambda}}, \qquad -\infty < x < \infty;\ \mu \in R; \lambda > 0$$

Find its m.g.f.

(b) Suppose the r.v. X has U(0,1) distribution . A random sample of size n is taken from it. Obtain the distribution of i)$X_{(1)}$ ii)$X_{(n)}$iii)Hence or otherwise find the distribution of range= R= $X_{(n)} - X_{(1)}$. **(08)**
