(3 hours) [Total marks: 60]

N.B.: (1) Attempt any four questions.
(2) Figures to the right indicate full marks.
(3) Simple non-programmable calculator is allowed.

1. (a) Let $\underline{X} = \left(\underline{X}^{(1)}, \underline{X}^{(2)}\right)' \sim N_p\left(\underline{\mu}, \Sigma\right)$. Derive the marginal distribution of $\underline{X}^{(1)}$. Derive the conditional distribution of $\left(\underline{X}^{(2)} \middle| \underline{X}^{(1)}\right)$. (8)

   (b) Let $\underline{X}_1, \underline{X}_2, \dots, \underline{X}_n$ be i.i.d $N_p\left(\underline{0}, \Sigma\right)$. Derive MLE of Σ. (7)

2. (a) Define Hotelling's T^2. Derive the Hotelling's T^2 statistic to test $H_0 : \mu = \mu_0$ Vs $H_1 : \mu \neq \mu_0$ when a random sample of size n is drawn from $N_p(\mu, \Sigma)$, $\Sigma > 0$ and unknown. (6)

   (b) Define multiple correlation coefficient $\boldsymbol{\rho}_{1.234\dots p}$. In usual notations show that $\boldsymbol{\rho}_{1.234\dots p}$ is the maximum correlation between X_1 and any other linear function of $X_2, X_3, \dots, X_p$. (6)

   (c) Obtain the distribution of $(X - \mu)' \Sigma^{-1} (X - \mu)$ when $X \sim N_p(\mu, \Sigma)$. (3)

3. (a) Let $\underline{X}_1, \underline{X}_2, \dots, \underline{X}_n$ be i.i.d $N_m\left(\underline{\mu}, \Sigma\right)$. Derive the LRT statistic λ to test the hypothesis $H_0: \underline{\mu} = \underline{0}, \Sigma = \Sigma_0$, where, Σ_0 is specified. Give the asymptotic distribution of $-2\ln\lambda$. (8)

   (b) Let $\underline{X}_i^{(k)}$ be an observation from $N_p\left(\underline{\mu}^{(k)}, \Sigma^{(k)}\right)$, $i = 1, 2, \dots, n_k, k = 1, 2, \dots, q$. Construct a test of the hypothesis $H_0: \Sigma^{(1)} = \Sigma^{(2)} = \cdots = \Sigma^{(q)}$. Find the exact distribution of the criterion for the case $p = 2$. (7)

4. (a) Describe Fisher's method for discrimination among several (say) populations assuming that the p x p population covariance matrices are equal and of full rank. Suppose a random sample of size n_i observations is available from π_i , $i = 1, 2, \ldots, g$ population then determine Fisher's sample linear discriminants. (10)

   (b) Discuss the procedure of testing the significance of the distance between two p- variate normal populations assuming that their variance matrices are equal. (5)

5. (a) Let $X \sim N_p(\mu, \Sigma)$. Consider $Y = AX$ where A is a kxp matrix of rank k (k < p). Prove that Y is distributed as a k-variate normal vector. (5)

   (b) Describe minimum ECM rule for k - class classification $(k > 2)$. (5)

   (c) Define Wishart distribution. If D is Wishart matrix following $W_p(f, \Sigma)$ and C is a matrix of order qxp with rank q then show that $CDC' \sim W_q(f, C\Sigma C')$. (5)
